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Stückelberg field-shifting quantization of a free particle on a D-dimensional sphere

C Neves† and C Wotzasek‡

† Centro Universitario de Barra Mansa, Barra Mansa, Rio de Janeiro, Brazil
and
Departamento de Física, ICE, Universidade Federal de Juiz de Fora, 36036-330, Juiz de Fora, MG, Brazil
‡ Instituto de Física, Universidade Federal do Rio de Janeiro, 21945-970, Rio de Janeiro, Brazil

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Abstract. In this paper we quantize a free particle on a D-dimensional sphere in an unambiguous way by converting the second-class constraint using the Stückelberg field-shifting formalism. Furthermore, we argue that this formalism is equivalent to the BFFT (Batalin–Fradkin–Fradkina–Tyutin) constraint conversion method and show that the energy spectrum is identical to the pure Laplace–Beltrami operator without additional terms arising from the curvature of the sphere. We work out the gauge symmetry generators with results consistent with those obtained through the nonlinear implementation of the gauge symmetry.

1. Introduction

The canonical quantization of the free-particle moving in a curved space is a fundamental theoretical problem that has been investigated intensively over the last few decades in different settings [1–4], but remains a controversial problem in the literature. The relevance of this problem to quantization on curvilinear surfaces is well appreciated and its quantization has been studied both in the path-integral and in the canonical approach. The quantum picture, however, remains troubled by operator ordering ambiguities [1] and the results following different approaches are not in complete agreement†. While most investigations have been aimed towards understanding the quantum nature directly from the second-class formulation, a possible loophole to avoid problems would be the reformulation of the model as a gauge theory.

The proposal of this paper is the construction of a gauge-invariant reformulation for the free point particle on the spherical surface through the Stückelberg field-shifting formalism [6]. This is possible after a nonlinear implementation of the Stückelberg symmetry through the elimination of the Lagrange multiplier sector of the invariant theory.

The treatment of nonlinear systems as gauge theories was originally proposed by Kovner and Rosenstein [7], using an analogy with quantum electrodynamics (QED) to disclose a symmetry hidden in the nonlinear sigma model (NLSM). An invariant version of this model was proposed by us [8] to explain the results of [7] using the iterative constraint conversion approach [9]. Recently, other authors [10–12] have proposed distinct first-class versions for the spherical model using the BFFT formalism [13]. Due to the possibility of dealing with

† For a review of the present status of this problem see [5].

the nonlinear constraint of the massive Yang–Mills theory through the constraint conversion technique, this problem has experienced a revival [14–16]. These works discuss the energy spectrum of the collective mode of the theory and call into doubt the result proposed by Adkins, Nappi and Witten (ANW) in [17].

It is worth mentioning that since the seminal work of Skyrme [18] incorporating baryons in the NLSM low-energy description of the strong interactions, the investigation of nonlinear theories has attracted much attention. The NLSM is a very useful model used in all areas of physics. In condensed matter, for instance, it is used to describe systems ranging from antiferromagnetic spin-chains to certain materials exhibiting the fractional quantum Hall effect [19]. In lower-dimensional physics, where it possesses an exact solution [20], it has became an important theoretical laboratory mainly due to its similarity to four-dimensional (4D) non-Abelian gauge theories with which it shares many features such as renormalizability, asymptotic freedom, dynamical mass generation, confinement and topological excitations. It has also been used in the theoretical investigation of the phenomenon of fractional spin and statistics in $(2+1)$D [21] and non-Abelian bosonization in $(1+1)$D [22].

In the study of the static properties of nucleons, carried out by Adkins *et al* [17], a collective semiclassical expansion is performed by the usual decomposition of the $SU(2)$ matrix into the nonlinear sigma model action as

$$U(r,t) = A(t)U(r)A(t)^{-1} \tag{1}$$

where the matrix $A(t)$ as $A(t) = a^o + \mathrm{i}a\tau$ satisfies the spherical constraint,

$$\phi_1 = a_i a_i - 1 = 0 \qquad \text{with} \quad i = 0, 1, 2, 3. \tag{2}$$

The theory becomes reduced to a nonlinear quantum mechanical model whose dynamics is governed by a Lagrangian dependent on $a_i(t)$ and $\dot{a}_i(t)$ playing the roles of the particle's coordinate and velocity, respectively. Similarly, the study of the fractional spin and statistics in the context of the $O(3)$ NLSM is reduced to that of the quantum rotor through the semiclassical separation of the collective mode, reducing the problem to that of quantizing the spherical top. Recall that the spherical rotor [4, 5] is the paradigm of the second-class constrained system with field-dependent Dirac brackets [23]. Therefore, the ambiguities resulting from the quantization of this model affect the above-mentioned results. This leads to the necessity of performing new studies that may eventually shed some light on these questions.

2. The spherical gauge model

To explore the problem that affects the quantization process for the nonlinear model, let us begin by quantizing the system using the Dirac method for second-class constraints. A free point particle with unitary mass moving on a flat $(D+1)$-dimensional Euclidean space is restricted to the D-spherical surface by the spherical constraint in configuration space,

$$q_i q_i - R^2 = 0 \tag{3}$$

where R is the radius of the sphere and $q_i(t)$, $i = 1, 2, \ldots, D$, are the coordinates of the particle. The dynamics of the point particle are governed by the Lagrangian

$$L = \tfrac{1}{2}\dot{q}^2 + \lambda(q_i q_i - R^2). \tag{4}$$

The corresponding Hamiltonian is

$$H = \tfrac{1}{2}p_i^2 - \lambda(q_i q_i - R^2). \tag{5}$$

The constraint analysis reveals the presence of four second-class constraints

$$\begin{aligned}\Omega_1 &= \pi_\lambda \\ \Omega_2 &= q^2 - c \\ \Omega_3 &= p_i q_i \\ \Omega_4 &= p_i p_i - 2\lambda q_i q_i .\end{aligned} \tag{6}$$

The geometrical meaning of Ω_2 and Ω_3 is transparent. The constraint $\Omega_2 = 0$ restrains the particle to move on the D-sphere surface, while Ω_3 means that the particle momentum remains tangential to a nonlinear surface, without a radial component during the motion. The remaining constraints, Ω_1 and Ω_4, have no geometrical meaning and dynamical importance in the theory since they are artefacts of constructing the Hamiltonian formalism from the Lagrangian using the Legendre transformation. This occurs because the Lagrange multiplier λ, which enforces the nonlinear constraint in the Lagrangian formalism, is assumed to be an independent dynamical variable. In this way, the Hamiltonian formalism yields these extra constraints to suppress the dynamics of λ and p_λ.

From the last condition of (6) the Lagrange multiplier can be computed explicitly as [10]

$$\lambda = \frac{1}{2}\frac{p^2}{q^2}. \tag{7}$$

The particle's dynamics can be described by $H = \frac{1}{2}p^2$ and two constraints (Ω_2, Ω_3). The symplectic structure on the physical phase space determined by these constraints is induced by the Dirac brackets,

$$\begin{aligned}\{q_i, q_j\}^* &= \{p_i, p_j\}^* = 0 \\ \{q_i, p_j\}^* &= M_{ij} \\ \{p_i, p_j\}^* &= H_{ij}\end{aligned} \tag{8}$$

where

$$\begin{aligned}M_{ij} &= \delta_{ij} - \frac{q_i q_j}{q^2} \\ H_{ij} &= \frac{(q_j p_i - q_i p_j)}{q^2}.\end{aligned} \tag{9}$$

It may be stressed that the same results can be obtained from the simplified Lagrangian formulation with the proviso that the equation of motion of the eliminated variable must be maintained as a subsidiary condition to impart consistency in the canonical analysis. Next we present a proposal to express it as a gauge theory using the Stückelberg field-shifting formalism.

In the literature there are alternative methods to implement this proposal. We quote the BFFT [13] and iterative [8, 9] methods that have attracted much attention in the literature. The BFFT conversion technique uses as many auxiliary variables as the number of second-class constraints [13]. As mentioned, the analysis can be considerably simplified by eliminating the multiplier sector of the phase space using the non-invariant character of the constraints [24]. The question that seems to be of importance is related to the elimination of this sector before or after the constraint conversion. The induced gauge symmetry over the spherical model becomes realized nonlinearly or linearly, respectively, leading to distinct consequences. In the former case, worked out in [10], the multiplier is eliminated before the BFFT procedure, based on its second-class character. On the other hand, without elimination of the multiplier sector, the gauge symmetry is linearly implemented by the Stuckelberg procedure. Although we advocate the latter procedure mostly because of its effectiveness and simplicity, we will show next that they indeed lead to (canonically) equivalent results.

Let us consider the construction of the Wess–Zumino (WZ) terms through the Stückelberg mechanism,

$$L_{WZ}(\theta,\lambda) = L(\lambda - \tfrac{1}{2}\dot{\theta}) - L(\lambda) \\ = \theta q_k \dot{q}_k. \tag{10}$$

An equivalent procedure using the iterative conversion of the nonlinear constraints was given in [8], whose gauge-invariant Lagrangian was found to be

$$L = \tfrac{1}{2}\dot{q}_i^2 + \theta(q\cdot\dot{q}) + \lambda(q^2 - R^2) \tag{11}$$

where θ is the WZ variable. The corresponding Hamiltonian, obtained reducing the Lagrangian (11) to first order, is

$$H = \tfrac{1}{2}p^2 + \tfrac{1}{2}\theta^2 q^2 - \theta(q\cdot p) - \lambda(q^2 - R^2). \tag{12}$$

This theory has two chains of constraints whose primary members are

$$\begin{aligned} \phi_1 &= \pi_\lambda \approx 0 \\ \psi_1 &= \pi_\theta \approx 0. \end{aligned} \tag{13}$$

Since these constraints must satisfy some integrability condition, the presence of secondary constraints is required, i.e.

$$\begin{aligned} \phi_2 &= q^2 - R^2 \approx 0 \\ \psi_2 &= q\cdot p - \theta q^2 \approx 0 \end{aligned} \tag{14}$$

and no more constraints are generated by following Dirac's algorithm. Although a naive inspection shows the presence of second-class constraints, the computation of the Dirac matrix shows the presence of two zero modes, indicating the existence of a two distinct set of constraints. One with two first-class constraints ($\varphi_k^{(1)}$) and other with two second-class constraints ($\varphi_k^{(2)}$), that are identified after a diagonalization of the Dirac matrix as

$$\begin{aligned} \varphi_1^{(1)} &= \phi_1 \\ \varphi_2^{(1)} &= \phi_2 - 2\psi_1 \end{aligned} \tag{15}$$

and

$$\begin{aligned} \varphi_1^{(2)} &= \psi_1 \\ \varphi_2^{(2)} &= \psi_2. \end{aligned} \tag{16}$$

The elimination of the second-class sector is done via the Dirac bracket reduction, as usual. It generates the following first-class Hamiltonian:

$$\bar{H} = \frac{1}{2}p_k\left(\delta_{km} - \frac{q_k q_m}{q^2}\right)p_m \tag{17}$$

where the reduced first-class constraints now read

$$\begin{aligned} \varphi_1^{(1)} &\to \bar{\varphi}_1^{(1)} = \phi_1 \\ \varphi_2^{(1)} &\to \bar{\varphi}_2^{(1)} = \phi_2. \end{aligned} \tag{18}$$

Similarly, the original Poisson brackets are now mapped into Dirac brackets,

$$\begin{aligned} \{q_i, q_j\}^* &= 0 \\ \{p_i, p_j\}^* &= 0 \\ \{q_i, p_j\}^* &= \delta_{ij}. \end{aligned} \tag{19}$$

Note that the Dirac brackets for this partial reduction of constraints have a canonical structure. This just reflects the result of the well known Maskawa–Nakashima theorem [25]. This new Hamiltonian and symplectic structure define a pure first-class problem. By simple inspection the correct equation of motion may be obtained from these objects. The symmetry transformations are generated by these constraints as $\delta_i \mathcal{O} = \varepsilon_i \{\mathcal{O}, \bar{\varphi}_i^{(1)}\}^*$ ($i = 1, 2$ and $\mathcal{O} = \lambda, q_k, p_k$),

$$\begin{aligned} \delta_1 \lambda &= \varepsilon_1 \\ \delta_1 q_i &= \delta_1 p_i = 0 \end{aligned} \tag{20}$$

and

$$\begin{aligned} \delta_2 \lambda &= 0 \\ \delta_2 q_i &= 0 \\ \delta_2 p_i &= -2\varepsilon_2 q_i. \end{aligned} \tag{21}$$

Note that the coordinates q_i are null eigenvectors of the matrix M_{ij}, defined in (9), acting as a phase space metric in the reduced Hamiltonian $\bar{H}$ in (17). To complete this discussion it is important to recall that the computation of the two sets of constraints given in (15) and (16) was imperative to the development of this procedure. However, the splitting computation of the original set of constraints may be obscure. To avoid this problem a systematic alternative, based on the reduction of the set of constraints with the elimination of the superfluous constraints is now elaborated that will illuminate the full power of the Stückelberg formalism.

Let us recall that the theory under discussion (11) is known to possess four constraints. However, for systems with holonomous constraints imposed by Lagrange multipliers, some of these constraints only appear in the canonical process to eliminate the dynamics associated with the multiplier sector of variables. It is usual practice to use an improved Hamiltonian obtained by eliminating the Lagrange multiplier sector *ab initio*. This will keep only the meaningful geometrical constraints and simplify the analysis. However, recall that the equation of motion associated with the (eliminated) multiplier sector is maintained as a consistency condition to the canonical structure associated with the simplified Lagrangian formulation. To eliminate the redundant constraints we proceed as follows. The Euler–Lagrange equations for θ and q_i, are solved,

$$\begin{aligned} q \cdot \dot{q} &= 0 \\ \ddot{q}_i + \dot{\theta} q_i - 2\lambda q_i &= 0 \end{aligned} \tag{22}$$

respectively, which determine the Lagrange multiplier as

$$\lambda = -\frac{1}{2q^2}(\dot{q}^2 - \dot{\theta} q^2). \tag{23}$$

The arbitrariness present in the multiplier reflects the gauge freedom induced over the system. Bringing this relation back into the WZ theory we find a new canonical structure given by the modified Hamiltonian,

$$\tilde{H} = \frac{1}{2R^2} q^2 p^2 - \frac{q^2}{R^2}(q \cdot p)\theta + \frac{1}{2R^2}\theta^2 (q^2)^2 \tag{24}$$

and the first-class, strongly involutive constraint

$$\omega_1 = q^2 - R^2 - 2\pi_\theta \approx 0 \tag{25}$$

which has no time evolution since,

$$\dot{\omega}_1 = \{\omega_1, \tilde{H}\} = 0. \tag{26}$$

On the other hand, consistency with the first equation in (22) requires π_θ to have no time evolution of its own,

$$0 = \dot{\pi}_\theta = \{\pi_\theta, \tilde{H}\}. \tag{27}$$

This condition imposes a new constraint over the system as

$$\omega_2 = q \cdot p - \theta q^2. \tag{28}$$

This canonical structure is similar to that obtained in [10] and identical to that given in [26], after a convenient interchange the WZ variables ($\theta \rightleftharpoons \pi_\theta$) with a corresponding change of signs.

Note that after the elimination of the multiplier λ some aspects of the model have changed. Here, π_θ is not a constraint but it is a canonical variable of the model that is absorbed by the nonlinear constraint, deforming the original spherical surface and destroying the constraint hierarchy given in (13) and (14). Consequently, after the exceeded constraints are eliminated, the remaining geometrical ones, and the gauge-invariant model described by the Hamiltonian (24) are equivalent to the original first-class system given in (12). This reduced gauge-invariant model has two first-class constraints ω_1 and ω_2, that obey the strongly involutive algebra,

$$\begin{aligned} &\{q^2 - R^2 - 2\pi_\theta, \tilde{H}\} = 0 \\ &\{q \cdot p - \theta q^2, \tilde{H}\} = 0 \end{aligned} \tag{29}$$

which is in agreement with the issues of [10, 26]. These first-class constraints generate the following infinitesimal gauge transformations on the canonical variables in the complete extended space:

$$\begin{aligned} \delta_1 q_i &= \varepsilon_1 \{q_i, \omega_1\} = 0 \\ \delta_1 p_i &= \varepsilon_1 \{p_i, \omega_1\} = -2\varepsilon_1 q_i \\ \delta_1 \pi_\theta &= \varepsilon_1 \{\pi_\theta, \omega_1\} = 0 \\ \delta_1 \theta &= \varepsilon_1 \{\theta, \omega_1\} = -2\varepsilon_1 \end{aligned} \tag{30}$$

and

$$\begin{aligned} \delta_2 q_i &= \varepsilon_2 \{q_i, \omega_2\} = \varepsilon_2 q_i \\ \delta_2 p_i &= \varepsilon_2 \{p_i, \omega_2\} = -\varepsilon_2 (p_i - 2\theta q_i) \\ \delta_2 \pi_\theta &= \varepsilon_2 \{\pi_\theta, \omega_2\} = \varepsilon_2 q_i^2 \\ \delta_2 \theta &= \varepsilon_2 \{\theta, \omega_2\} = 0 \end{aligned} \tag{31}$$

that agrees with those obtained in [7, 8, 10, 26]. The finite induced WZ gauge symmetries within the extended phase space are obtained from the gauge generating constraints by successive application on the canonical and the extended phase space variables,

$$\begin{aligned} &q_i \to \mathrm{e}^{\varepsilon_2} q_i \\ &p_i \to \mathrm{e}^{-\varepsilon_2} p_i + 2q_i (\theta \mathrm{e}^{\varepsilon_2/2} - (\theta + \varepsilon_1)\mathrm{e}^{-\varepsilon_2/2}) \\ &\theta \to \theta - 2\varepsilon_1 \\ &\pi \to \pi + (1 - \mathrm{e}^{-\varepsilon_2}) q^2. \end{aligned} \tag{32}$$

Note that the group of transformations generated by the first-class constraints act nonlinearly on the extended phase space.

We stress that Kovner–Rosenstein's hidden symmetry is indeed an induced symmetry of the Wess–Zumino sector over the phase space of the theory. This effect, as discussed above is clearly independent of the particular method of constraint conversion, being quite unique. Indeed, the constraint ω_1 in (25) is immediately transformed into the KR constraint generator with a special value for π_θ. Interestingly, this also corresponds to a choice of initial condition in (27). This is revealed by gauge-fixing the WZ sector in such a way as to recover the spherical constraint as the symmetry generator of the KR symmetry. Either way, this may be achieved by adding the gauge-fixing constraint,

$$\omega_3 = \pi_\theta \tag{33}$$

to the set ω_1 and ω_2. The $\Omega = \omega_1|_{\pi_\theta}$ constraint now plays the role of a Gauss law generator for the KR symmetry in the original phase space, under the Dirac bracket algebra generated by the second-class constraints ω_2 and ω_3. This reduced algebra has the same canonical structure as in (19) which is another illustration of the Maskawa–Nakashima theorem [25]. The dynamics is controlled by the Hamiltonian (24) which on the constraint shell $\omega_i \approx 0$ reads

$$H_{KR} = \frac{1}{2R^2} q^2 p_k \left(\delta_{km} - \frac{q_k q_m}{q^2} \right) p_m \tag{34}$$

which is seen to be the one postulate in [7]. This purely first-class Hamiltonian structure leads to the correct field equations under the induced Dirac bracket algebra.

To realize the quantization it is necessary to introduce a gauge-fixing term in order to fix the first-class nature of the Gauss law. Choosing the gauge condition as

$$\Psi = p_D = 0 \tag{35}$$

which is the canonical momentum conjugate to the coordinate q_D and removes the dynamic of this coordinate. The Poisson brackets between the constraints Ω and Ψ is

$$\{\Omega, \Psi\} = 2q_D \tag{36}$$

and as $q_D \neq 0$ on the spherical surface, they form a set of second-class constraints and the theory passes to having two-dimensional remaining phase space variables. The Dirac brackets among the independent variables are

$$\begin{aligned} \{q_\alpha, q_\beta\}^* &= 0 \\ \{q_\alpha, p_\beta\}^* &= \delta_{\alpha\beta} \\ \{p_\alpha, p_\beta\}^* &= 0 \end{aligned} \tag{37}$$

where α and β represent the independent phase space variables. The non-invariant Hamiltonian in the reduced phase space is

$$H = \frac{1}{2R^2} p_\alpha g^{\alpha\beta} p_\beta \tag{38}$$

with the non-singular phase space metric,

$$g^{\alpha\beta} = \delta^{\alpha\beta} - \frac{q^\alpha q^\beta}{R^2}. \tag{39}$$

This Hamiltonian formulation of the problem has also been found by Abdalla and Banerjee [5] following a purely second-class approach to quantize the system. In the following we follow

[5] closely in order to find the spectrum of the skyrmion. Since this system is unconstrained the velocities obtained from the Hamiltonian equation of motion for q_α,

$$\dot{q}_\alpha = g^{\alpha\beta} p_\beta \tag{40}$$

can be obtained in an unambiguous form from the canonical momenta by inverting the above equation,

$$p_\beta = g_{\alpha\beta}\dot{q}_\alpha \tag{41}$$

where $g_{\alpha\beta}$ is the inverse of (39), given by

$$g_{\alpha\beta} = \delta_{\alpha\beta} + \frac{q_\alpha q_\beta}{R^2 - q^2}. \tag{42}$$

In the quantization of nonlinear models the ordering of phase space fields cannot be neglected since the Dirac brackets are field dependent, as carried out in [27]. Therefore, there arises an important question as to how one should settle the quantum Hamiltonian from its corresponding classical description. The answer resides on the preservation of the classical symmetries in the quantum scenario [3]. In this way the corresponding quantum Hamiltonian is uniquely determined. Based in the quantum process developed in [3] the quantization of the reduced nonlinear model (38) is accomplished if the reduced Hamiltonian is replaced by the corresponding Laplace–Beltrami operator defined as

$$\begin{aligned}\hat{H} &= -\tfrac{1}{2} g^{-1/2} \partial_\alpha g^{\alpha\beta} g^{1/2} \partial_\beta \\ &= -\tfrac{1}{2} (R^2 - q^2)^{-1/2} \partial_\alpha g^{\alpha\beta} (R^2 - q^2)^{1/2} \partial_\beta\end{aligned} \tag{43}$$

where $\partial_\alpha = \frac{\partial}{\partial q_\alpha}$ are the derivatives with respect to the D-dimensional curved space coordinates, and g is the determinant of the metric $g_{\alpha\beta}$ given by

$$\begin{aligned}\det[g_{\alpha\beta}] &= \exp \operatorname{tr} \ln \left(\delta_{\alpha\beta} + \frac{a_\alpha a_\beta}{R^2 - q^2} \right) \\ &= \exp \operatorname{tr} \frac{q_\alpha q_\beta}{q^2} \ln \left(1 + \frac{q^2}{R^2 - q^2} \right) \\ &= \frac{R^2}{R^2 - q^2}.\end{aligned} \tag{44}$$

Due to this, the Hamiltonian operator (43) is related to the angular momentum in the reduced space,

$$\begin{aligned}L_{\alpha\beta} &= q_\alpha p_\beta - q_\beta p_\alpha = -\mathrm{i}\hbar (q_\alpha \partial_\beta - q_\beta \partial_\alpha) \\ L_{\alpha D} &= q_\alpha p_D - q_D p_\alpha = -\mathrm{i}\hbar q_D \partial_\beta = -\mathrm{i}\hbar (R^2 - q^2)^{1/2} \partial_\beta\end{aligned} \tag{45}$$

and therefore it is rewritten as

$$\hat{H} = \sum_{\alpha\beta} \frac{L_{\alpha\beta}^2}{2R^2}. \tag{46}$$

Thus, we find that the quantum Hamiltonian is the conventional Schrödinger operator without any extra curvature term. Consequently, the energy spectrum reads

$$E = \frac{1}{2R^2} l(l + D - 1) \tag{47}$$

in agreement with the result obtained by other authors [5, 28–30].

At this stage it is interesting to put our result in a more realistic framework that might shed some light over the question. To this end we focus our discussions on the *Skyrme model*. There $D = 3$ and consequently, the energy spectrum (47) becomes

$$E = \frac{1}{2R^2} l(l+2) \tag{48}$$

that agrees with the result proposed by ANW [17]. This completes our discussion.

3. Conclusion

In summary, the gauge symmetry of the nonlinear model is induced by phase space extension methods using the Stückelberg field-shifting constraint conversion, displaying the equivalence with the constraint conversion methods. Afterwards the energy spectrum was obtained without an additional constant term arising from the curvature of the D-sphere. Subsequently, the *Skyrme model* was considered to study this scenario and the energy spectrum was also obtained without extra terms.

To conclude this section it is important to give some views concerning the reduction process for the multiplier sector: whether it is reduced before or after the constraint conversion leads to distinct realizations of the WZ symmetry. We have verified that the procedure of reduction commutes with the constraint conversion process, leading to results which are canonically equivalent. This seems to be of importance for the analysis of non-Abelian second-class systems as gauge theories where quadratic constraints are intrinsically defined. Finally, it becomes clear that the question regarding the construction of the generators of the WZ gauge symmetry cannot be tackled from this approach, in the sense that there is no plausible argument that favours any of the constraints as the leader of the constraint chain. If this question becomes an important issue for the analysis of the problem at hand then the use of the non-Abelian BFFT method or the iterative constraint process seems unavoidable.

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